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AN APPROXIMATE PLUME ANALYSIS FOR THE VOYAGER TASK C PLANETARY QUARANTINE STUDY

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AN APPROXIMATE PLUME ANALYSIS FOR THE VOYAGER TASK C PLANETARY QUARANTINE STUDY

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TABLE OF CONTENTS

SECTION 1
INTRODUCTION

An important mission constraint within the Voyager Program is the Planetary Quarantine requirement.

A survey of potentially important causes of contamination has been carried out, and this survey indicated that the various propulsion systems associated with the spacecraft may prove to be significant contamination sources via their exhaust gases. The exhaust gases themselves may prove to contain viable microorganisms. However, even if the exhaust environment is found to be lethal to any microorganisms in the propellants, the exhaust gases may still "sweep" non-sterile spacecraft surfaces causing the release of possible contaminants.

Quantitative assessment of the contamination probability associated with these possible sources of contamination requires definition of the plume characteristics for the propulsion and altitude control gas systems.

In this report we consider the problem of determining the distribution and velocity of solid particles in the plume flow field of a jet expanding into vacuum. The problem divides itself naturally into two parts. We must first determine the plume flow field and then consider the trajectory of small particles which are coupled to the plume dynamics.

In Section 2 we first consider the structure of the plume flow field. As we shall see, the centerline behavior of the jet may be, for some conditions, adequately modeled by a source flow. We discuss the results for the centerline quantities and in addition consider the flow-field off-axis.

In Section 3 we utilize the results of Section 2 to consider the trajectories of small, micron sized particles in the jet flow. We arrive at results for their number density, and velocity as a function of particle size. In general, the results indicate that viable organisms and micron sized solid particles will be confined within a 30° cone about the nozzle centerline.

The analysis undertaken in this report does not include the back flow portion of the plume. However, further consideration will be given to the effects of the back flow portion of the plume when the lander recontamination problem is analyzed.

SECTION 2
PLUME FLOW FIELD ANALYSIS

The theoretical aspects of the plume problem are quite intriguing. In the absence of physical boundaries, we have a flow which undergoes a transition from collision-dominated isentropic expansion in the vicinity of the nozzle exit plane, to a collision-free or free molecular flow far downstream from the nozzle. This sequence of events has been established experimentally in several laboratories; the measurements show that the Mach number along the centerline of the jet increases according to a collision-dominated isentropic expansion until collisions are too infrequent to support the expansion. The Mach number increase is then observed to level off and the expansion is said to "freeze"; this is in direct contradiction to the continuum, isentropic theory which predicts a continual rise.

In order to model the complex three-dimensional free jet flow the problem of one-dimensional, spherical source flow has been considered by Hamel and Willis.[1] As pointed out by Sherman,[2] the source flow model will probably be a quite good representation of events along the jet centerline, but will become progressively worse as the source Reynolds number decreases. In this section we discuss the usefulness of the source flow model for centerline predictions and summarize calculation of flow quantities along the centerline. In addition, we consider possible corrections to it when source flow is no longer valid, also the flow field off-axis is discussed and simple formulas for quantities off axis are presented.

2.1 SOURCE FLOW MODEL FOR CENTERLINE QUANTITIES

As a problem in gas dynamics, source flow has been examined quite extensively. Most investigators have either considered flow from a line source or a spherical source and, with the exception of Brook and Oman,[3] have used the Navier-Stokes equations as their point of departure. Although Brook and Oman do proceed from the Bhatnagar-Gross-Krook kinetic equation, they neglect some of the terms on the left hand side of the equation written in spherical coordinates. The moment equations which result from this over simplified form of the B.G.K. equation are correct only for the conservation of mass. Their equation does not conserve momentum or energy and is grossly inaccurate in the derivation of the higher moment equations. Their results, therefore, are unlikely to be of more than qualitative value.

Without engaging in polemics about the advisability of using the Navier-Stokes equations to describe transition flow, we can certainly agree that the Boltzmann equation and kinetic theory will contain all the necessary details of continuum, transition and free-molecular flow (at least for momatomic gases). The point of view of Hamel and Willis has therefore been to develop a self-consistent approximation to the Boltzmann equation which is appropriate to the hypersonic, rarefied flow in the expanding jet and so avoid the controversies about the relative merits of Navier-Stokes equations and the various ad hoc moment approaches. The fundamental idea being that instead of establishing the approximation on the magnitude of the collision frequency (and Knudsen number), either near-continuum or near free molecular, we only demand that the Mach number or Speed ratio of the flow (ratio of the ordered velocity to the thermal velocity) be large compared to one. The use of this hypersonic approximation allows a self-consistent truncation of the moment equations. We now summarize and discuss the principal results obtained for the source flow expansion and indicate how they may be applied to calculation of the free jet quantities.

Summary of Variables

$$\underline{r} = \underline{r}' / r'_*$$

$$T = T' / T'_s$$

$$V = V' / \sqrt{\frac{kT'_s}{m}}$$

$$\alpha = \frac{2}{3}\left(\frac{\rho^{*\prime}}{\rho'_s}\right)\frac{V^{*\prime}}{\sqrt{\frac{kT'_s}{m}}}\left(\frac{\Pi}{2}\right)^{1/2}\frac{r'_*}{\lambda'_s} \qquad (1)$$

where primed quantities are still taken as dimensional and the subscript s refers to stagnation values and, sonic values. Following Sherman (for $\gamma = 5/3$) we can define α in terms of the variable most commonly used by free-jet experimenters:

$$\alpha = 0.6068 \frac{r'_*}{\lambda'_s} = 0.413 \frac{D'}{\lambda'_s} = 0.413 \, (Kn_D)^{-1}$$

where D' is the diameter of orifice from which the jet issues.

Scaling

$$r = a_1 \, x$$

$$T_{\parallel} = b_1 \, y_o$$

$$T_{\perp} = b_1 \, z_o$$

$$b_1 = (4.0395)^{3/(3+4\beta)} \; \alpha^{-4/(3+4\beta)} \qquad (2)$$

$$a_1 = \frac{(4.0395)^{3\beta/(3+4\beta)}}{5} \; \alpha^{3/(3+4\beta)}$$

where $\beta = 0$ for Maxwell molecules and 1/2 for hard sphere molecules. For a high temperature rocket exhaust $\beta \approx 1/2$. The temperature of the gas is then $T = \frac{1}{3} T_{\parallel} + \frac{2}{3} T_{\perp}$.

Summary of Results

We first write for the terminal Mach numbers (for $x \gg 1$)

$$M_{\parallel} = \frac{3^{1/2} \; \alpha^{2/(3+4\beta)}}{\left[y_o \, (\infty) \right]^{1/2} (4.0395)^{3/(6+8\beta)}}$$

$$\frac{M_{\perp}}{M_{\parallel}} = \frac{x^{1/2} \sqrt{2} \, (1+c)^{\beta/2}}{\left[Y_o \, (\infty) \right]^{\beta/2}}$$

where $\bar{y}_o \; (\infty)$ is of order one for all force laws. We now list the numerical values of $y_o \; (\infty)$ in tabular form for the limiting cases of Maxwell molecules ($\beta = 0$) and psuedo-hard sphere molecules ($\beta = 1/2$) and c = 0 and 2.

β	c	$\left[y_o(\infty)\right]_{\text{Numerical}}$
0		1.290
1/2	0	0.850
1/2	2	1.080

(4)

and

β	c	$M_{\parallel\infty}$
0	-	$0.421\ Kn_D^{-0.667}$
1/2	0	$0.868\ Kn_D^{-0.4}$
1/2	2	$0.770\ Kn_D^{-0.4}$

Having listed the asymptotic values of $H_{\parallel}$ and $M_{\perp}$ we now list some analytical approximations for $T_{\parallel}$ and $T_{\perp}$ which fit the computed profiles for all values of r, within 5%. These approximations represent expansions for small x and large x.

For $\beta = 1/2$, $c = 0$:

$$x \leq 1$$

$$y_o\, x^{4/3} = 1 + 1.014\, x^{3/5} - 0.200\, x^{6/5} \tag{5}$$

$$x \geq 1$$

$$y_o = 0.850 + 0.810\,\frac{1}{x} + 0.244\,(1/x)^2$$

where $T_{\parallel} = b_1\, y_o\,(x)$.

For $z_o\,(x)$, the isentropic solution $z_o\,(x) = x^{-4/3}$, describes $z_o\,(x)$, for $x \leq 1$, within 12% so that it can be considered as a good approximation in this region.

For $\beta = 1/2$, $c = 0$, $x \geq 1$:

$$z_o\,(x) = 0.530\left(\frac{1}{x}\right) + 0.437\left(\frac{1}{x}\right)^2 \tag{6}$$

and we compute T from: $T = b_1\, z_o\,(x)$.

We now consider how one would apply these results to the calculation of free jet centerline flow quantities under conditions of large source Reynolds numbers (i.e. $\alpha >> 1$):

(i) the density of the gas may be considered to be written as:

$$n = (r^{-2})\, n^*$$

(ii) the velocity along the centerline will be very nearly constant after about six nozzle diameters downstream of the exit plane. It's value will be:

$$V_\infty = \sqrt{5}\left(\frac{2\, k\, T_s}{m}\right)^{1/2}$$

(iii) the temperature of the gas along the centerline will be given by the formulas in eqs. (5) thru (6) where

$$T = \frac{1}{3}\, T_{\parallel} + \frac{2}{3}\, T_{\perp}$$

and a_1 and b_1 can be calculated.

The question which now arises is when does the source flow model breakdown for calculation of centerline quantities. We can make an estimate of this and set forth a rational procedure for correcting the results of this section in this limit. We consider below a sketch of the free jet:

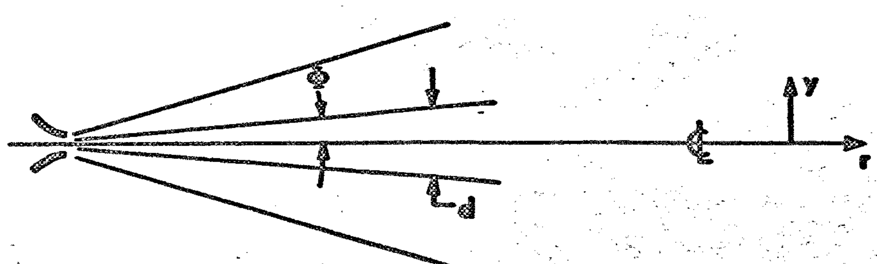


Figure 1

As the gas expands in the jet, the density increases radially along centerline; in addition, gradients of density exist transverse to the streamlines, and the density eventually goes to zero as one moves away from the axis. If we consider the width of the jet (i.e. that distance, y, from the axis for a constant density plateau, i.e. 90% of its value on the centerline), d, then a quantity $Kn_{\perp} = \frac{\lambda \Phi}{d}$ may be defined. It is clear that if $Kn_{\perp} \approx 1.0$; the gas will no longer be collision dominated transverse to the axis. If $Kn_{\perp}$ takes on a unity value on the same length scale that rarefaction effects occur for the centerline expansion then the centerline expansion may no longer be modeled by a simple source flow. We can state this quantitatively by writing:

$$Kn_{\perp} = \left(\frac{r^*}{d}\right) \alpha^{-1} \; r^2 = (\alpha \tan \Phi)^{-1} \; r \tag{7}$$

since $\frac{d}{r^*} = r \tan \Phi$, where $\tan \Phi \approx .2$, the density profile off-axis is given by the formula,

$$\frac{n\ (r,y)}{n\ (r,o)} = \cos^2 \theta \ \cos^2 \frac{\pi \theta}{2.8}$$

so that to find Φ we can solve the equation:

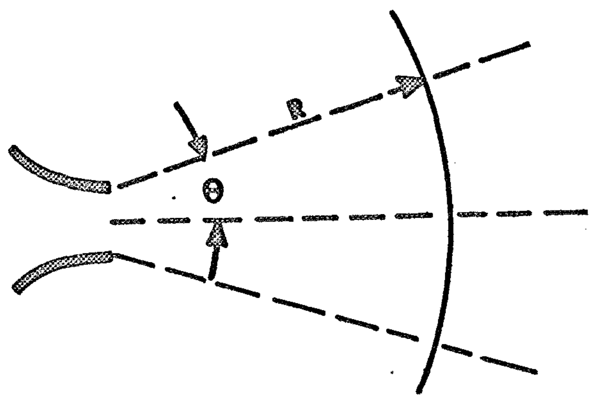
$$.9 = \cos^2 \Phi \ \cos^2 \frac{\pi \Phi}{2.8}$$

Experimental results of Muntz indicate that when $Kn_{\perp} = .2$ at the middle of the transition regime, signifcant non-source flow effects occur. For helium, Muntz finds that for $\alpha \geq 125$ source flow is a good approximation for centerline quantities. The non-source flow effects manifest themselves as a lowered collision frequency on centerline and consequently earlier (closer to the source) rarefaction effects in the free jet. For $\alpha \leq 125$ modification of the Hamel and Willis source flow theory should be possible to take account of non-source flow effects on the free-jet centerline.

2.2 OFF-AXIS CONSIDERATIONS

As mentioned earlier, the unmodified Hamel and Willis source flow theory should be adequate for centerline predictions for $\alpha \geq 125$. Off the centerline, the density profile as correlated by Ashkenas and Sherman [5] is:

$$\frac{n\ (R,\ \theta\ )}{n\ (R,\ 0\ )} = \cos^2\left(\frac{\pi\ \theta}{2.8}\right) \tag{8}$$



To find the freeze temperature off-axis we may simply consider that for each streamline a new effective α may be written as:

$$\alpha_{eff.} = \alpha\left(\cos^2\left[\frac{\pi\ \theta}{2.8}\right]\right) \tag{9}$$

The $\alpha_{eff.}$ written above may then be substituted for α into the formulas written earlier for the simple source flow. The main assumptions here are that the directions of the streamlines are formed in the inviscid regime and are essentially source like with the limiting velocity:

$$V_\infty = \sqrt{5}\ \left(\frac{2\ k\ T_s}{m}\right)^{1/2}$$

Using the formulas for the density and the $\alpha_{eff.}$ of eq. (8), it should be possible for a large source Reynolds number to estimate the off-axis properties of the flow field.

SECTION 3
PARTICLE DYNAMICS

In the previous sections we have considered approximate procedures for calculating the structure of the free jet expansion. In this section we consider the distribution of particles which travel from the nozzle exit plane through the expanding jet flow.

The exhaust gases that leave the nozzle expand into the vacuum of space and undergo a considerable turning. If we were to consider the flow from the nozzle as a source flow:

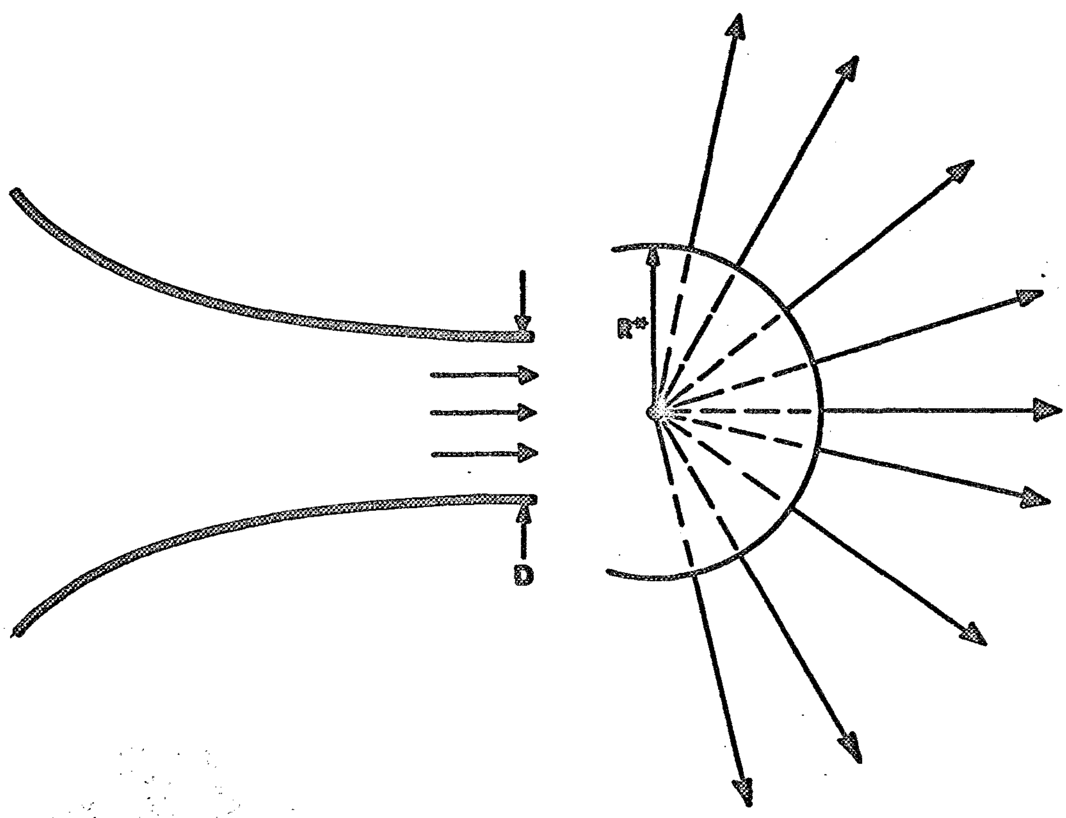


Figure 2

that is to say, within a few nozzle diameters of the exhaust plane, the flow will seem to be source-like, emanating from a spherical source of radius, $R^* = .75\ D$. The parallel exhaust plane streamlines are therefore turned by the expanding flow within a relatively short distance from the nozzle and they emanate with constant angle rays from an apparent source. The particles (and here we may distinguish between spore like macromolecules and micron sized particles) do not, however, turn with the streamlines, since they are not in perfect momentum contact with the lighter carrier gas. To analyze the motion of each solid particle would involve a rather large scale computing program which would solve the equations of motion for each particle of a given size and subject to certain initial conditions. For engineering purposes results such as these are difficult and expensive to obtain and one would rather have simple correlation formulae which give approximate information about all classes of particles.

The spirit of this analysis is then to investigate the phenomena and the equations that govern them, and so obtain approximate relations for particle positions and velocity which will not require laborious and expensive computer computation.

We now assume that the particles at the nozzle exit plane have a uniform profile of velocity and number density and that this profile is unchanged for several nozzle diameters so that at the source flow radius we have a uniform stream of particles which then interact with the carrier gas and are turned through some angle. To write this mathematically, we write for the particles an equation of motion:

$$m_p \frac{d \underline{V}_p}{dt} = \underline{F}_{gp} \qquad (10)$$

where $\underline{F}_{gp}$ represents the turning force exerted on the particle by the carrier gas flow. To approximate $\underline{F}_{gp}$ we chose to say that:

$$\underline{F}_{gp} = m_g \nu_{gp} (\underline{V}_p - \underline{V}_g) f \qquad (11)$$

where ν_{gp} is a collision frequency for the particle with gas molecules, m_p the mass of the particle and $\underline{V}_p$, and $\underline{V}_g$ the velocity of the particle and gas respectively.

The form used for $\underline{F}_{gp}$ is indeed difficult to choose since we require it to be correct in the limit where we consider spore-like macromolecules and in addition where we consider micron size particles. To account for this we introduce a factor f which tells us how many molecules may simultaneously collide with a particle. That is, f is the ratio of the effective molecular radius of the gas molecule to the radius of the particle. So that if $f \approx 1$, we consider only one gas molecule and one particle interactly instantaneously, while if $f \gg 1$, there are many collisions and the particle will be in free molecular flow. For intermediate f we have a Brownian motion. We in addition now assume that the effect of the momentum transfer which most concerns us is the turning of the particles so that $|\underline{V}_p| = V$ exit.

We can then write for the particle an equation (comparable to (11)) which describes the angular turning:

$$\frac{d\phi_p}{dt} = \left(\frac{m_g}{m_p}\right) \nu_{gp} \left[\phi_p - \phi_{p\,eq.}\right] \qquad (12)$$

where ϕ_p is the angular position of particle (See Figure 3), and $\phi_{p_{eq.}}$ the position it would have if it followed the streamlines of the gas. The size of the particle will essentially come into the expression for f

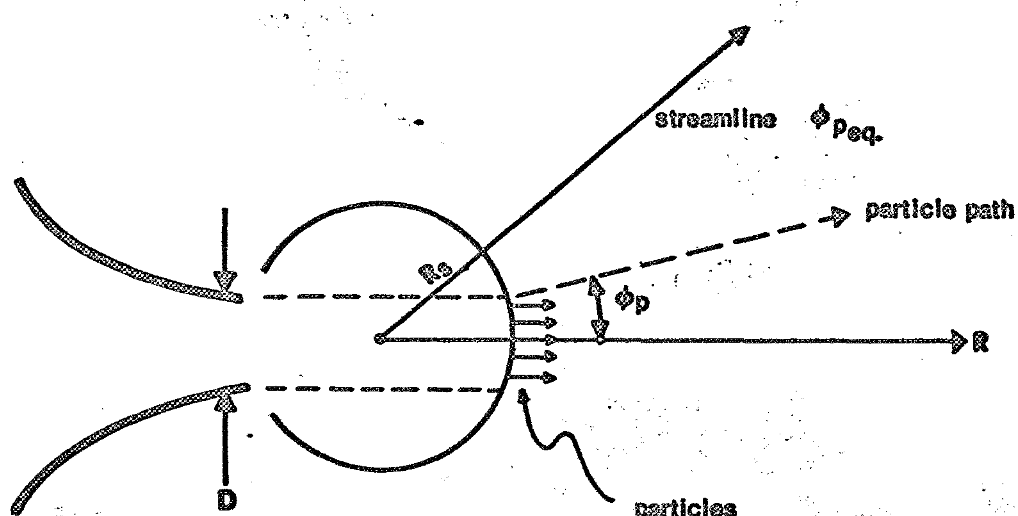


Figure 3

We now solve eq. (12) with $\phi_p - \phi_{p\,eq.}$ taken at its largest exit value.

$$\phi_p = f\left[\frac{\gamma}{V\,\text{exit}}\int_{R^*}^{R} \nu_{gp}'\,(R')\;dR'\right]\left[\cos^2\left(\frac{\pi\,\phi_{p\,eq.}}{2\,\phi_0}\right)\right](\phi_{p\,eq.}) \qquad (13)$$

where $\nu_{gp} = \nu_{gp}'\,(R')\,\cos^2\left(\frac{\pi\,\phi_{p\,eq}}{2\,\phi_0}\right)$

$$\phi_{p\,eq.} \leqslant \tan^{-1}\frac{D}{2R^*}$$

and $\nu_{gp}' = \left(\frac{\mu\ \text{viscosity}}{V\,\text{exit}\ \ P\,\text{exit}}\right)\left(R'\right)^{-2}, \quad \phi_0 = \frac{\pi}{3}$

and we let $\gamma = \frac{m_g}{m_p}$

$$\phi_p = f\left(\frac{\gamma}{V^2\,\text{exit}}\right)\left(\frac{\mu\ \text{viscosity}}{P\,\text{exit}}\right)\left[\frac{1}{R^*}-\frac{1}{R}\right]\left(\phi_{p\,eq.}\right)\cos^2\left(\frac{\pi\,\phi_{p\,eq}}{2\,\phi_0}\right) \qquad (14)$$

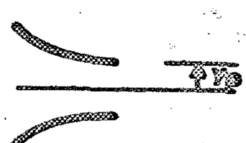
That is to say, if we consider a particle at the exit plane it will have a certain $\phi_{p\,eq.}$ associated with it and so eq. (14) will then prescribe the final angle ϕ_p that it will achieve. We can compactly work for $R \rightarrow \infty$:

$$\phi_{p\,final} = f\left(\gamma\,a\right)\left(\phi_{p\,eq.}\right)\cos^2\left(\frac{\pi\,\phi_{p\,eq}}{2\,\phi_0}\right) \qquad (15)$$

where, $a = \left(\frac{\mu\ \text{viscosity}}{p\,\text{exit}\ \ V^2\,\text{exit}}\right)\frac{1}{R^*}$

Again, $f = \frac{r_p}{r_m}, \qquad \gamma = \frac{m_g}{m_p}$.

Typically, $a = 10^4$ and $\phi_{p\,eq.}$ is related to the position of the particle in the exit plane of the nozzle by relation:



$$\phi_{p\,eq} = \tan^{-1}\frac{Y_p}{R^*}$$

where Y_p is the position of the particle. In the above expressions r_p is the particle radius and r_m the molecular radius. With little approximation we may say the particle velocity and temperature remain essentially equal to their exit plane values (neglecting radiative cooling).

We finally, for purposes of estimation, will show that for both spores and micron size particles, the turning angle will be small. For a spore: $f \cong 1$, $\gamma \approx 5 \times 10^{-6}$ and so ϕ_p final $\approx 5 \times 10^{-2}$ (ϕ_p eq.). While for a particle $f \approx 10^8$, $\gamma \approx 10^{-14}$ and ϕ_p final $\approx 10^{-2}$ ϕ_p eq. These estimates show that all particles, including really small spores, will be confined to within say 15° of the centerline, and for engineering purposes this will be a reasonable assumption.

SECTION 4
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SECTION 5

NOMENCLATURE

a_1	See equation (2)
b_1	See equation (2)
c	See equation (3)
d	distance from axis, See Figure 1.
f	ratio of particle radius to molecular radius: r_p / r_m
F_{gp}	force on particle
k	Boltzmann Constant
m	molecular weight
m_p, m_g	mass of particle, mass of a molecule of gas
n	number density
n*	number density at exit plane
$\underline{r}$	position coordinate
r_p, r_m	radii of particle and molecule
R	distance from nozzle - See Figure 3.
R*	source radius - - See Figure 2.
T	temperature
$T_{\parallel}$, $T_{\perp}$	parallel and perpendicular temperature
T_s	source temperature
$\underline{V}$	velocity
V_∞	velocity of gas far from the nozzle
Vp	particle velocity
V_{exit}, V*	both refer to exit plane velocity
X	See equation (2)
Y	See Figure 1.
Yo	See equation (2)

z_o	See equation (2)

<u>Greek Symbols</u>

α	See equation (1)
β	See equation (2)
γ	$\gamma = mg/mp$
Θ	See Figure 1.
Φ	See Figure 1.
ϕp	angular position of particle
ϕp eq.	angular position of streamline
νgp	collision frequency, See equation (13)
λ	mean free path
λs	mean free path in the source
$\lambda_{\mathbb{C}}$	mean free path on the jet centerline